Solution to problem “billiard”

Team of Germany (Jan Theofel*)

12th IYPT in Vienna

This paper presents the solution to the problem number 11 “billiard” of the 12th IYPT in Vienna by the German team. It consists of a simulation with additional theoretical ideas. Thanks to Mr. Luding of the IAC1 from the university of Stuttgart for the simulation, to Martin Trautmann for his work on this problem and to the complete German preparation group for their ideas.

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1 Introduction

1.1 The original problem

Before a pool-billiard game starts, 15 balls form an equilateral triangle on the table. Under what conditions will the impact of the white ball (16th ball) produce the largest disorder of the balls?

This was the problem no. 11 of the 12th IYPT which was hold in Vienna in 1999. For more informations about the IYPT see [1] to [3].

The general idea of the tournament is that one team presents a solution to one of the problems which are published about half a year before the IYPT starts. A second team is the opponent, who must find the good ideas in the solution but also the mistakes. Afterwards both teams will get in a discussion about the solution presented and when it’s finished, the reviewer will make a short summary of everything.

It is somehow important to know that I did never present this solution within the tournament. I only was opponent against the USA in the 2nd selective fight.

1.2 Our basic ideas

To solve this problem, we took a few steps. At first, we made some definitions about the size of the table, the mass of the balls, etc. The values used are according to the official rules.

The next thing is to define how disorder is defined. This is very important of course, because without a good definition all the rest of the work is useless. To be on the save side, we decided to choose four different methods and to compare between them.

As a database for the search of the best hit we decided to use a computer simulation. This is useful, because real experiments would take too much time and it would be hard to measure the disorder.

2 Definition of the problem

2.1 Disorder at the end

It seems to be the best to concentrate on the disorder when all balls have stopped after the hit again. Of course also the moment directly after the impact of the white ball into
the triangle could be looked at. But there are however some good reasons to take the final state.

It is much easier to measure and it is also much easier to find a useful definition of disorder. Another reason for it is, that for a player only the final state is interesting, because he will do his next hit, when all balls have stopped.

2.2 The chosen table and balls

The values for the billiard balls were taken according to international rules.

![Diagram of the table]

Figure 1: The table

2.2.1 The balls

For the diameter of the balls we took 5.2 cm, the density is \( \frac{3 \text{ kg}}{1} \). This results in a mass of 220 g per ball.

2.2.2 The table

As the table we decided to take one, which we could also use for experiments in reality. The sizes can be taken from figure 1.

The width of the table is 1.2 m and the length is the double, i.e. 2.4 m. One fourth of the length (0.6 m) is the distance for the white ball starting line and the upper ball of the triangle.
3 The simulation

3.1 Running the simulation

The main simulation which calculates one hit was written in FORTRAN by Mr. Luding from the IAC1 of the university of Stuttgart. A small PERL script I wrote myself generates the initial files and runs the simulation.

For the final results generation another program written in C++ by Martin Trautmann is used. It collects all final states and calculate the disorder. The values are written to a text file and displayed with gnuplot.

To be able to run so many different hits (≈ 25000) we did some parallel processing on 10 computers for a few days. For further simulations we will use even more. If you have some free calculation time for more of our simulations and you have a Linux-/UNIX-based system, please contact me. (see 5.2)

3.2 Features of the simulation

The simulation includes 16 equal balls of which 15 form at the beginning an equileral triangle. Energy is lost by roll friction and hits of the balls against other balls or the wall.

A small script runs the simulation with different starting positions, angles and speeds of the white ball. In order to save calculation time, we have only chosen starting positions on one side of the table, because for symetry reasons the others are the same (in the simulation). Most of the starting positions are close to the middle of the table, because it is most common to start there.

For every startposition angles between -50° and 50° were simulated (where negativ angles go to left and positiv to the right side). For the middle and the position directly at the wall we only used angles starting at 0°. The difference of angle was 0.25° for three of the different positions, for the others 1° was taken.

In order to get realistic speed for the white ball, we have chosen speeds between 1 and $5 \frac{\text{m}}{\text{s}}$ in steps of $0.5 \frac{\text{m}}{\text{s}}$. But for lower speeds the ball did not hit the triangle. This might indicate that our roll friction was too strong and should perhaps be corrected.

The output of the simulation comes in text files. It is therefore very easy to handle and can also be read by humans which is very useful although it makes the program a little bit slower. While the simulation is running, there is no graphical output which makes it faster again. For visualization please see section 3.4. If you are interested in these files, please read section 5.2.

3.3 Simplifications of the simulation

As we only want to get hits which produce in general a great disorder, we made some simplifications for the simulation.

The first one is that the table does not include any pockets. This simplification can be accepted, when we remind that for the first hit mostly no balls get in the pockets. I will come back to that later.
It also does not include rotational energy. In reality this energy causes that the ball which hit another will not handle over all it’s energy to the hiten one. This was not implemented realistically, but the hit are not really elastic.

As another simplification we left out the spin of the balls. This can be done in our opinion, because it is not so important in ball–ball interactions but only in ball–wall interactions. It will be mentioned later.

### 3.4 Visualization by xballs

For visualizing the hit we used a programm called xballs which was also written by Mr. Luding.

![Figure 2: The xball visualization](image)

The colors of the balls indicate the energy they have, i.e. their speed because they only have kinetic energy. A small line on them shows us the direction the are rolling just now.

In the down right corner some important values are shown. For us only the time is important. The graphs on the right and the bottom show the density of balls and the density of energy. This could also be the basis of a disorder definition but we did not use it therefore.

It is also possible to draw a grid with xballs to make the boxcount better visible.
4 Methods to measure disorder

We have calculated different methods, because we think none of them is the best. But they all together make conclusionable results.

4.1 Direct methods

4.1.1 Linear distance

Here we sum up all the distances between the balls. As higher the value is, as higher the disorder, because the balls fill the table better.

4.1.2 Square distance

This is nearly the same as linear distance, but the distances are squared before adding them. By this change longer distances count much stronger and short ones will nearly drop out.

4.1.3 Box count

Here we divided the table in boxes. (Each box was 20 cm times 20 cm.) The value for the disorder is the number of boxes which “got” a ball divided by the total number of boxes.

4.1.4 Stretched box count

This is very like box count, but the boxes get bigger from the one side where at the beginning the triangle is to the side where the white ball starts. We thought this could also be a good method, because it’s obvious, that in the upper half of the table (where the white ball starts) not so many balls will stop after the first hit. So this method works against this phenomenon to get a balanced system again.

4.2 Indirect Methods

4.2.1 Difference between two similar hits

It would be interesting to compare very similar hits. But this wasn’t done yet, because the difference between two of our hits is too big for this method in our opinion.

4.2.2 Duration of the process

The shorter the process goes, the better is the disorder! Surprised? When the systems needs less time to get stopped by rolling friction, that means that the energy is very well distributed to all balls, so the disorder should be good.
5 Further research and contacts

5.1 Further research

This project is – in my opinion – not yet finished. There are a lot of more hits which can (and will) be simulated. This is one important aspect to collect more information about the problem. Also different tables should be simulated.

The explanations for a lot of phenomenon are not yet found. This is probably the most important point for further research. As long as some of the disorder values are not explained, this project can’t be called finished.

Also some other methods of measuring disorder should be added. In the end they could all be put together to one value which indicates the disorder.

Last but not least the simulation must get better. The processes which are already there must become more realistic. Also the pockets, spin and so on have to be added to the simulation.

5.2 Contacting the author

On my website http://www.theofel.de/iypt/ you can find more about this problem. There are the original slides I used for the tournament, the graphics as gif files and the results of the simulation for your own work.

If you want to make suggestions, send your ideas or questions, please mail me at jan@theofel.de.