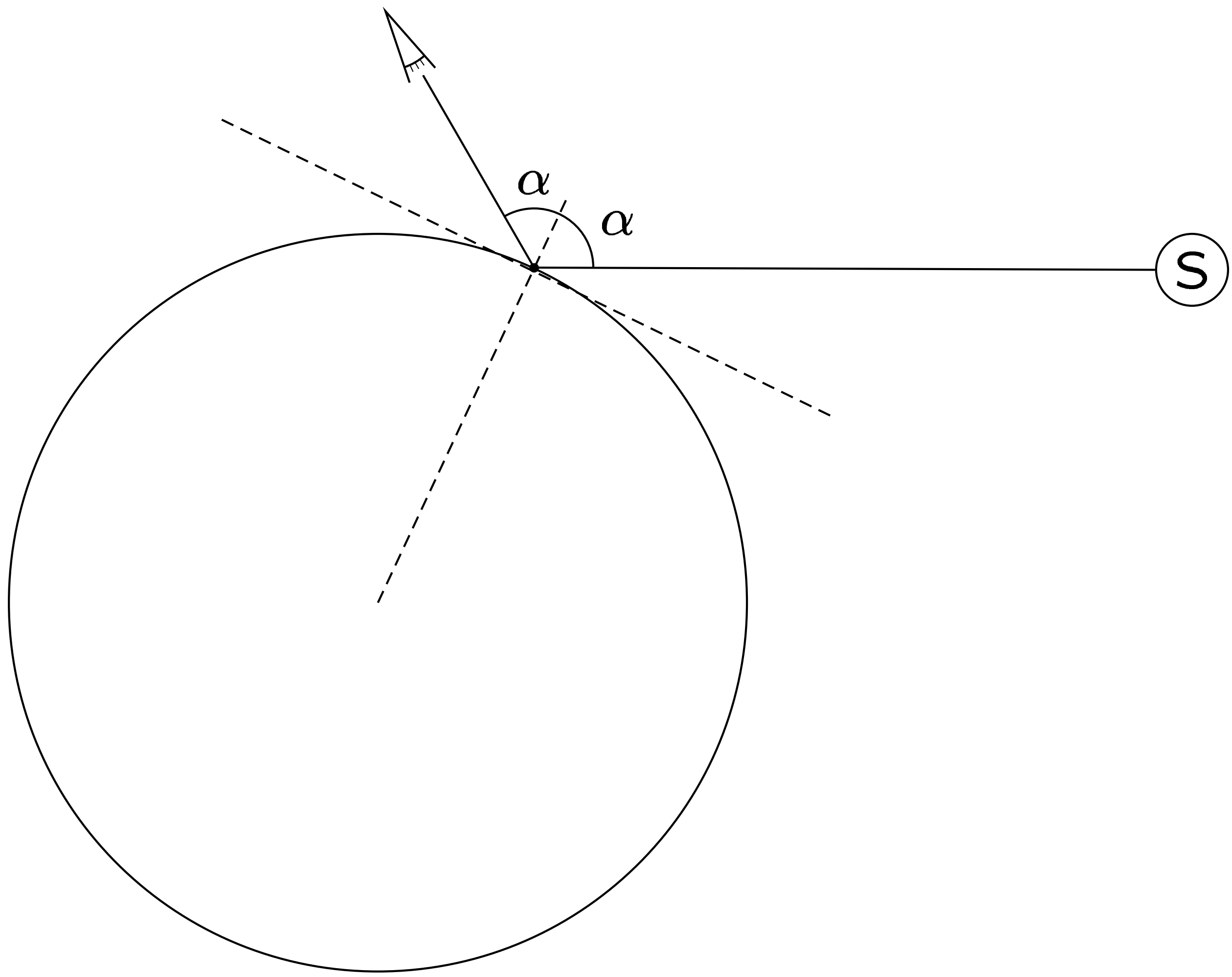
Bright Spots

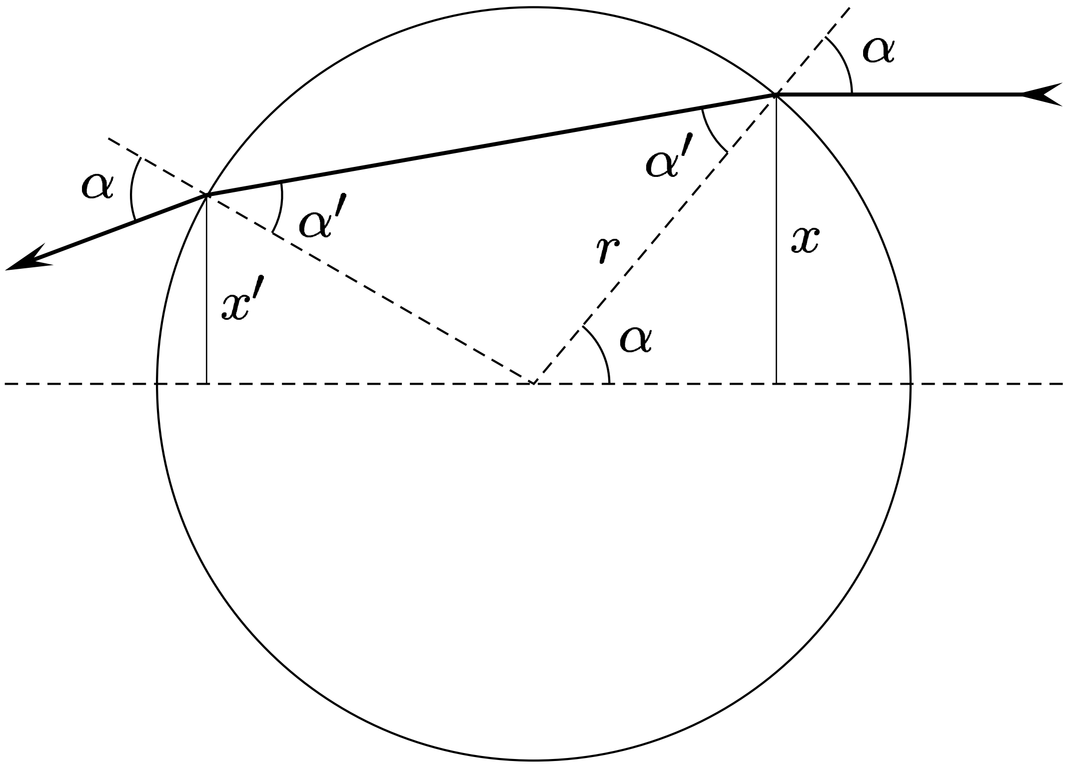
Problem:
Bright spots can be seen on dew drops if you look at them from different angles.
Discuss this phenomenon in terms of the number of spots, their location and angle of observation.

- general ideas
- raytracing
- experiments
- comparison between theory and reality

The spot of *1^{st} order* is the direct reflection of the sunray.



The spot of *2^{nd} order* is refracted twice.



general ideas (2^{nd} order)

$$\xi = \frac{x}{r} = \sin\alpha$$

$$\xi' = \frac{x'}{r} = \sin\left(2\alpha' - \alpha\right)$$

$$\sin\alpha = n\sin\alpha'$$

$$\xi' = \sin(2\alpha')\cos\alpha - cos(2\alpha')\sin\alpha$$

$$= \frac{2}{n}\xi\sqrt{1-\left(\frac{\xi}{n}\right)^2}\sqrt{1-\xi^2} - \left(1 - 2\left(\frac{\xi}{n}\right)^2\right)\xi$$

two special cases:

$$\xi' = 0 \text{ for } \xi = 0 \quad \text{(direct ray of light)} \tag{1}$$

$$\xi' = 0 \text{ for } \xi > 0 \tag{2}$$

$$\sqrt{1-\left(\frac{\xi}{n}\right)^2}\sqrt{1-\xi^2} = \frac{n}{2}\left(1-2\left(\frac{\xi}{n}\right)^2\right) \tag{3}$$

$$1-\frac{n^2}{4} = \frac{\xi^2}{n^2} \tag{4}$$

$$\xi = n\sqrt{1-\frac{n^2}{4}} \tag{5}$$

with $n = 1.33$ (for water) we get $\xi \approx 0.99$
$n = 1.6$ (for glass) follows $\xi \approx 0.96$

for the extreme value:

$$\xi = 1 \quad \text{for} \quad x = r \tag{6}$$

$$\xi' = \frac{2}{n^2} - 1 \tag{7}$$

with $n = 1.33$ (for water) results $\xi' \approx 0.13$

We used PovRay$^{(tm)}$ for creating pictures of the phenomenon.

Mostly **two spots** can be observed, all other spots are less bright and hardly to see.

The number of spots which can be seen easily depends only on the reflection and refraction coefficients.

The angle of observation doesn't play an important role. Only for a few angles the number of spots is reduced.

experiments

- walk around a drop

- laserlight on glass shperes

- light on “artificial” drops in the dark

In nature mostly two spots can be observed.

The 1^{st} is less bright than the 2^{nd}.

Sometimes one of the points can be invisible because of different shapes of the drops on plants.

More than two spots can also be observed sometimes. Under good conditions can be seen up to ten spots (laboratory cases).

$$
\begin{aligned}
& \frac{2}{n}x\sqrt{\left(1-\left(\frac{x}{n}\right)^2\right)\left(1-x^2\right)}-\left(1-2\left(\frac{x}{n}\right)^2\right)x \\
= & \frac{2}{n}x\sqrt{1-x^2-\frac{x^2}{n^2}+\frac{x^4}{n^2}}-x+2\frac{x^3}{n^2} \\
= & \frac{2}{n}x\sqrt{1-x^2\left(1+\frac{1-x^2}{n^2}\right)}-x+2\frac{x^3}{n^2} \\
\approx & \frac{2}{n}x\left[1-\frac{x^2}{2}\left(1+\frac{1-x^2}{n^2}\right)\right]-x+2\frac{x^3}{n^2} \\
= & x\left[\frac{2}{n}-\frac{x^2}{n}-\frac{2}{n^3}+\frac{x^2}{n^3}-1+\frac{2x^2}{n^2}\right] \\
= & x\left(\frac{2}{n}-\frac{2}{n^3}-1\right)+x^3\left(\frac{1}{n}+\frac{2}{n^2}+\frac{1}{n^3}\right)
\end{aligned}
$$

In der Herleitung ist x das, was im restlichen Vortrag ξ ist!!!

Die Taylor-Näherung gilt nur für kleine $x^2\left(1+\frac{1-x^2}{n^2}\right)$. Es gilt:

$$\sqrt{1+\varepsilon} \approx 1+\frac{\varepsilon}{2}$$

$\varepsilon = 0.3 \quad \Rightarrow \quad 1+\frac{\varepsilon}{2} = 1.15 \approx 1.14 = \sqrt{1+\varepsilon}$ Das entspricht einem x (bzw. ξ) von 0.46.