

Dropped Paper

Problem:

If a rectangular piece of paper is dropped from a height of a couple of meters, it will rotate around its long axis whilst sliding down at a certain angle. What parameters does the angle depend on?

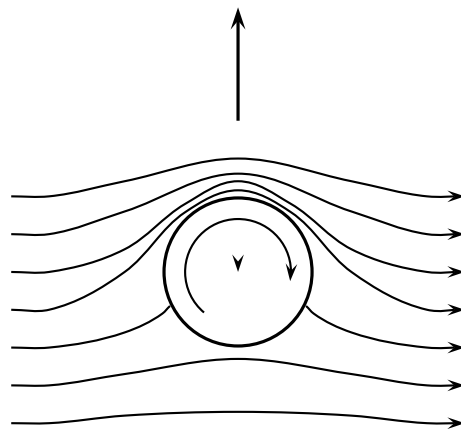
- Theory
 - * Autorotation of the paper
 - * Magnus effect
 - * Determination of the angle
- Experiment
- Conclusions

Autorotation of the paper

- autorotation depends on the Reynoldsnumber
- direction of rotation depends on start angle

Magnuseffect and buoyancy

- **Magnuseffect:** A rotating wing put in a stream creates a force on itself perpendicular to the stream in the direction of the rotation

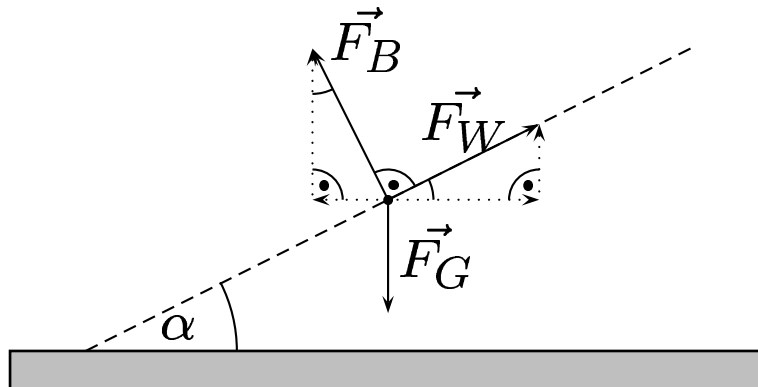


- **buoyancy** can be calculated with Bernoulli equation:

$$F_B = 2\rho_{\text{air}}v_0\omega RA$$

Determination of the angle

- 3 forces work on paper:
 - gravity
 - buoyancy
 - air resistance



linear movement

- after certain time paper is flying nearly on one line with a constant velocity
- sum of forces then is 0
- solution of that brings:

$$\begin{aligned}\sin \alpha &= \sqrt{1 - \frac{4\rho_{\text{air}}^2 \omega^4 R^6 l^2}{c_w^2 m^2 g^2} - \frac{2\rho_{\text{air}} \omega^2 R^3 l}{c_w m g}} \\ &= \sqrt{1 - \frac{\rho_{\text{air}}^2 \omega^4 R^4}{c_w^2 \sigma^2 g^2} - \frac{\rho_{\text{air}} \omega^2 R^2}{c_w \sigma g}}\end{aligned}$$

- ρ_{air} – density of the air
 ω – angular velocity
 R – half width of the paper
 l – length of the paper
 σ – mass per area
 g – gravitational acceleration

- approximate values for measurements:

$$c_w \approx 0.85$$

$$(R = 1 \text{ cm}, l = 8.5 \text{ cm}, \alpha = 45^\circ, \omega = 100 \text{ Hz}, m = 1 \text{ g})$$

Buoyancy

$$p + \rho_{\text{air}}gh + \frac{\rho_{\text{air}}}{2}v^2 = \text{const.}$$

$$v_1 = v_0 + v_R$$

$$v_2 = v_0 - v_R$$

$$p_1 + \rho_{\text{air}}gh_1 + \frac{\rho_{\text{air}}}{2}v_1^2 = p_2 + \rho_{\text{air}}gh_2 + \frac{\rho_{\text{air}}}{2}v_2^2$$

$$R \ll h$$

$$h_1 \approx h_2$$

$$p_1 + \frac{\rho_{\text{air}}}{2}(v_0 + v_R)^2 = p_2 + \frac{\rho_{\text{air}}}{2}(v_0 - v_R)^2$$

$$\Delta p = \frac{\rho_{\text{air}}}{2} [(v_0 - v_R)^2 - (v_0 + v_R)^2]$$

$$\Delta p = \frac{\rho_{\text{air}}}{2} [v_0^2 - 2v_0v_R + v_R^2 - v_0^2 - 2v_0v_R - v_R^2]$$

$$\Delta p = 2\rho_{\text{air}}v_0v_R$$

$$F_B = 2\rho_{\text{air}}v_0\omega RA$$

Determination Of The Angle

$$F_G = F_W \sin \alpha + F_B \cos \alpha$$

$$F_W \cos \alpha = F_B \sin \alpha$$

$$F_G = F_W \sin \alpha + F_W \frac{\cos^2 \alpha}{\sin \alpha}$$

$$F_W = F_G \sin \alpha$$

$$\frac{1}{2} c_w \rho_{\text{air}} A v_0^2 = mg \sin \alpha$$

$$\sin \alpha = \frac{c_w \rho_{\text{air}} A v_0^2}{2mg}$$

$$F_G = F_B \frac{\sin^2 \alpha}{\cos \alpha} + F_B \cos \alpha$$

$$F_B = F_G \cos \alpha$$

$$v_0 = \frac{mg \cos \alpha}{2 \rho_{\text{air}} \omega R^2 l}$$

Determination Of The Angle

$$\sin \alpha = \frac{c_w \rho_{\text{air}} m^2 g^2 \cos^2 \alpha A}{8 m g \rho_{\text{air}}^2 \omega^2 R^4 l^2}$$

$$\frac{\sin \alpha}{\cos^2 \alpha} = \frac{c_w m g}{4 \rho_{\text{air}} \omega^2 R^3 l}$$

$$C = \frac{c_w m g}{4 \rho_{\text{air}} \omega^2 R^3 l}$$

$$\frac{\sin \alpha}{\cos^2 \alpha} = C$$

$$C - C \sin^2 \alpha - \sin^2 \alpha = 0$$

$$\sin^2 \alpha + \frac{\sin \alpha}{C} = 1$$

$$\sin \alpha = -\frac{1}{2C} + \sqrt{1 - \frac{1}{4C^2}}$$

$$\sin \alpha = \frac{\sqrt{4C^2 - 1} - 1}{2C}$$

$$\sin \alpha = \frac{\sqrt{c_w^2 m^2 g^2 - 4 \rho_{\text{air}}^2 \omega^4 R^6 l^2} - 2 \rho_{\text{air}} \omega^2 R^3 l}{c_w m g}$$

$$\sin \alpha = \sqrt{1 - \frac{4\rho_{\text{air}}^2 \omega^4 R^6 l^2}{c_w^2 m^2 g^2} - \frac{2\rho_{\text{air}} \omega^2 R^3 l}{c_w m g}}$$

with $m = 2Rl\sigma$

$$\Rightarrow \sin \alpha = \sqrt{1 - \frac{\rho_{\text{air}}^2 \omega^4 R^4}{c_w^2 \sigma^2 g^2} - \frac{\rho_{\text{air}} \omega^2 R^2}{c_w \sigma g}}$$