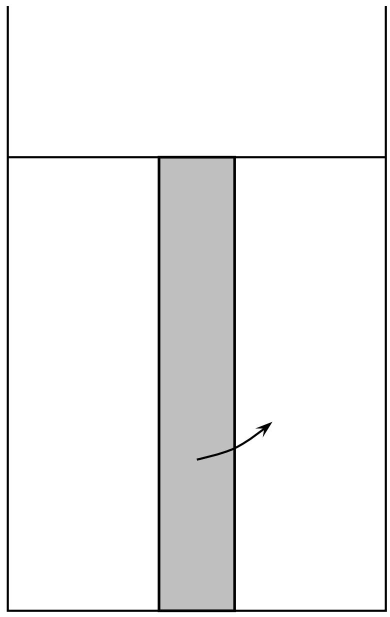
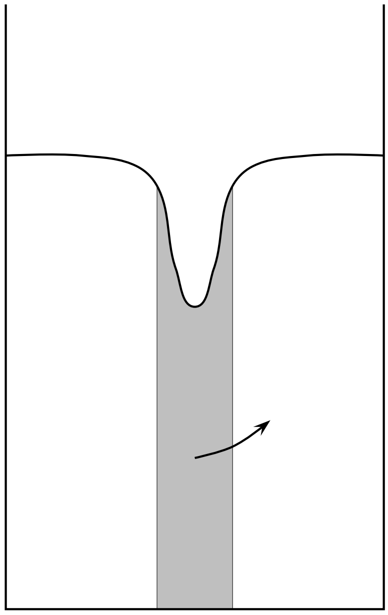
Rotation

Problem:
A long rod, partially and vertically immersed in a liquid, rotates about its axis. For some liquids this causes an upward motion of the liquid on the rod and for others, a downward motion.
Explain this phenomenon and determine the essential parameters on which it depends.

- theory
  * downward motion
  * upward motion
- experiments
- conclusions

In the middle of rotating liquids a **spinal column** can be found. It rotates like a solid body.

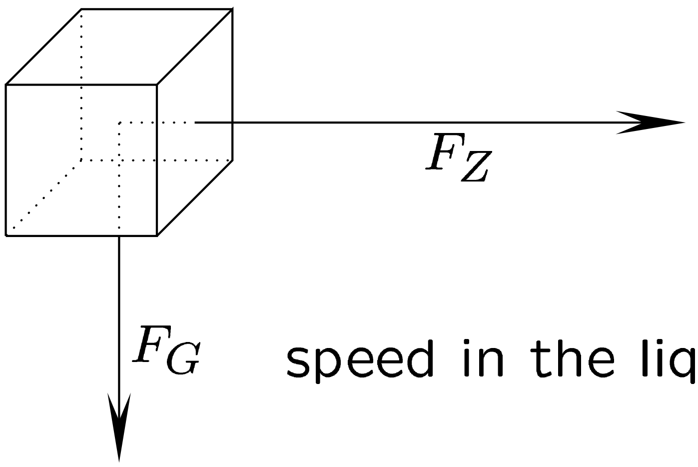




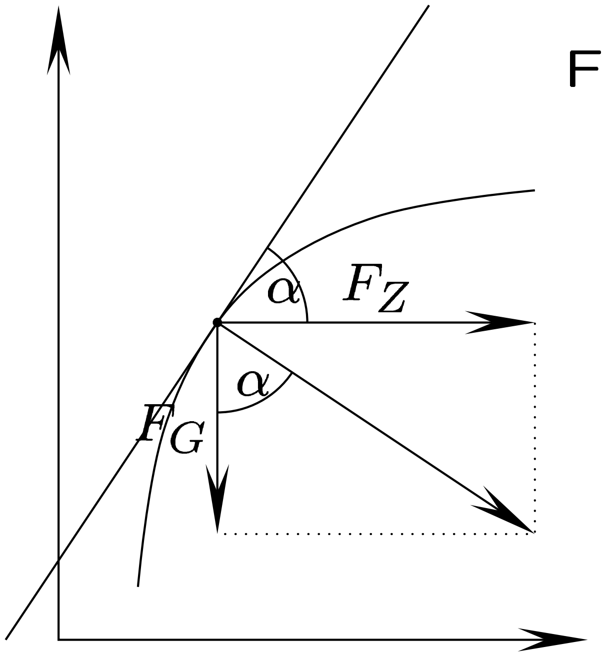
In this area the downward motion would have the form of $y \sim x^2$.

When rotating a rod in a liquid this **area is replaced by the rod**. Therefore the outer area must be observed.

It has the following form (for infinite case):



$$\text{speed in the liquid} : v_L \sim \frac{1}{r}$$
$$v_L = \frac{c}{r} \quad (1)$$
$$\text{centrifugal force} : F_Z = m\frac{v_L^2}{r} \quad (2)$$
$$\text{gravity} : F_G = mg \quad (3)$$



For the forces results:

$$\tan\alpha = \frac{F_Z}{F_G} = \frac{dy}{dr} \qquad (4)$$

$$dy = \frac{c^2 dr}{r^3 g}$$

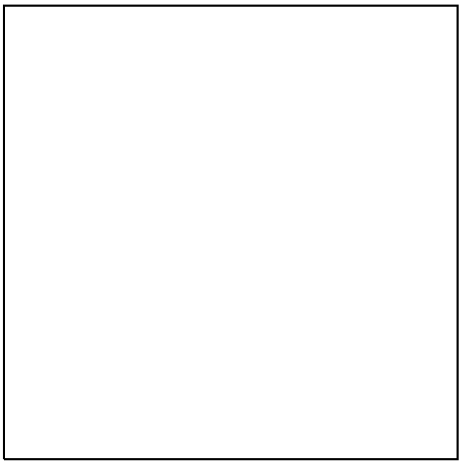
$$y = \int \frac{c^2 dr}{r^3 g}$$

The shape of the liquid surface is therefore:

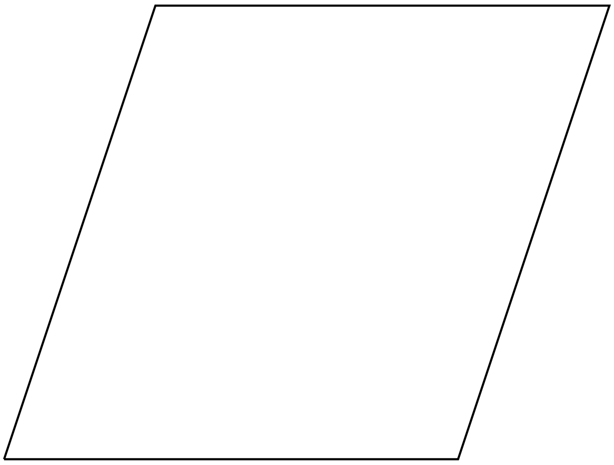
$$\boxed{y = -\frac{c^2}{g}\frac{1}{2r^2}} \qquad (5)$$

theory: upward motion (1/4)

Polymer liquids are totally different:

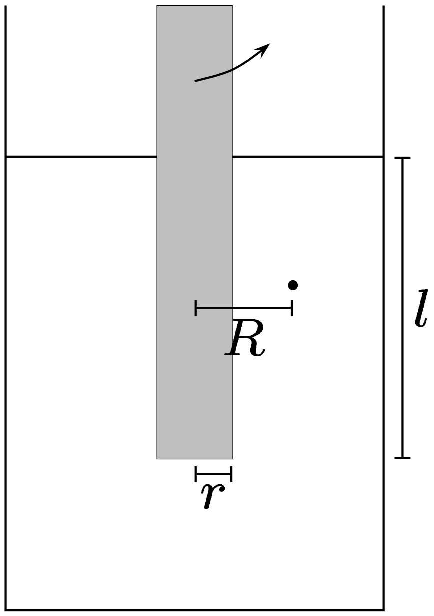


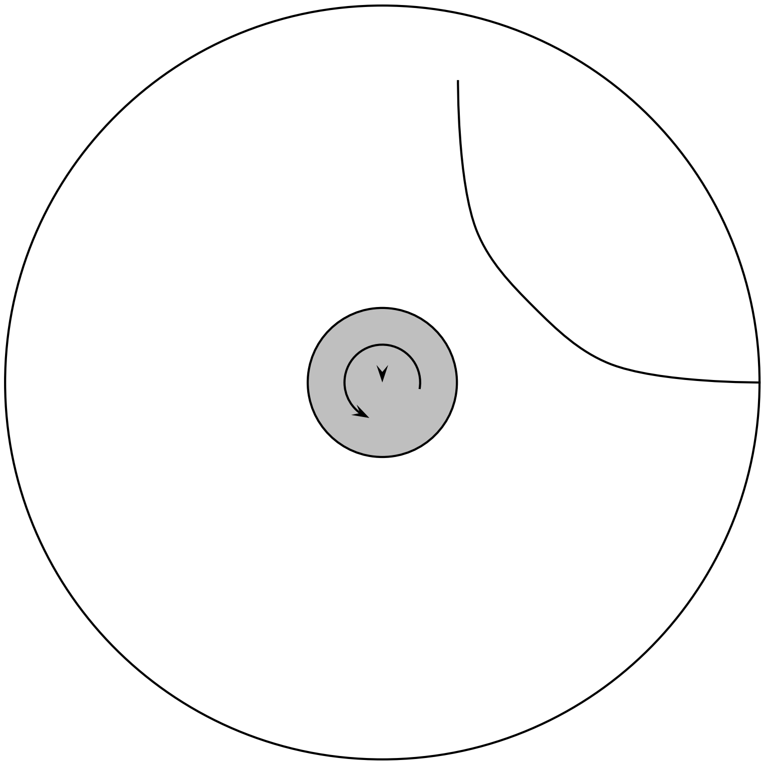
coiled up as fluffy little ball



in sheared liquids

theory: upward motion (2/4)





- velocity profile
- comparison: rubber band
- shear rate $\dot{\gamma}$ generates the shear stress

$$S = \eta\dot{\gamma} \tag{6}$$

- creates the torque

$$T = 2\pi R^2 l S \tag{7}$$

principle of mechanics:

$$2\pi r^2 l S_{\text{rod}} = 2\pi R^2 l S_R \tag{8}$$

with (6) follows:

$$r^2 \dot{\gamma_{\text{rod}}} = R^2 \dot{\gamma} \tag{9}$$

with $\dot{\gamma_{\text{rod}}} = 2\omega$ (literature) follows

$$\dot{\gamma} = 2\omega \frac{r^2}{R^2} \tag{10}$$

with the material constant Ψ follows for the normal stress

$$N = 4\Psi\omega^2 \frac{r^4}{R^4} \tag{11}$$

energetic assumption $W = pV \Rightarrow dW = \underbrace{pdV}_{\to 0} + Vdp$:

$$
\begin{aligned}
Vdp &= Nh2R\pi dR \\
-R^2hdp &= Nh2R\pi dR \\
dp &= -2\frac{N}{R}dR \\
dp &= -8\Psi\omega^2\frac{r^4}{R^5}dR \qquad (12) \\
p &= 2\Psi\omega^2\frac{r^4}{R^5} \qquad (13)
\end{aligned}
$$

with gravity follows:

$$
\begin{aligned}
\rho g h &= 2\Psi\omega^2\frac{r^4}{R^5} \\
h &= \frac{2\Psi\omega^2}{\rho g}\frac{r^4}{R^5} \qquad (14) \\
h_{\text{max}} &= \frac{2\Psi\omega^2}{\rho g} \qquad (15)
\end{aligned}
$$

Our polymer liquid was **Sedipur**.

For the experiments in **both cases** applies:

1st: They can't proof the theory.

But the measurement is difficult, because the effects are very little for easy reachable speeds.

2nd: But they harmonize with theory.

conclusions

- downward motion for **Newton liquids**
- upward motion for **polymer(-like) liquids**
- $\Rightarrow$ known as the **Weissenberg-effect**